M13/5/MATHL/HP3/ENG/TZ0/DM/M

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MARKSCHEME

May 2013

MATHEMATICS
DISCRETE MATHEMATICS

Higher Level

Paper 3

11 pages

1. (a) using the Euclidean Algorithm,

$332 = 3\times 99 + 35$ ***M1***

$99 = 2\times 35 + 29$ ***A1***

$35 = 1\times 29 + 6$

$29 = 4\times 6 + 5$ ***A1***

$6 = 1\times 5 + 1$ ***A1***

hence 332 and 99 have a gcd of 1 ***AG***

> **Note:** For both (a) and (b) accept layout in tabular form, especially the brackets method of keeping track of the linear combinations as the method proceeds.

[4 marks]

(b) (i) working backwards, ***(M1)***

$6 - 5 = 1$

$6-(29-4\times 6)=1$ or $5\times 6-29=1$ ***A1***

$5\times(35-29)-29=1$ or $5\times 35-6\times 29=1$ ***A1***

$5\times 35-6\times(99-2\times 35)=1$ or $17\times 35-6\times 99=1$

$17\times(332-3\times 99)-6\times 99=1$ or $17\times 332-57\times 99=1$ ***A1***

a solution to the Diophantine equation is therefore

$x=17$, $y=57$ ***(A1)***

the general solution is

$x=17+99N$, $y=57+332N$ ***A1A1***

> **Note:** If part (a) is wrong it is inappropriate to give ***FT*** in (b) as the numbers will contradict, however the ***M1*** can be given.

(ii) it follows from previous work that

$17\times 332 = 1 + 99\times 57$ ***(M1)***

$\equiv 1 \pmod{57}$ ***(A1)***

$z = 332$ is a solution to the given congruence ***(A1)***

the general solution is $332+57N$ so the smallest solution is 47 ***A1***

[11 marks]

Total [15 marks]

2. (a) (i) there is an Eulerian trail because there are only 2 vertices of odd degree ***R1***
there is no Eulerian circuit because not all vertices have even degree ***R1***

(ii) *eg* GBAGFBCFECDE ***A2***

[4 marks]

(b) (i)

Step	Vertices labelled	Working values	
1	A	A(0), B-3, G-2	***M1A1***
2	A, G	A(0), G(2), B-3, F-8	***A1***
3	A, G, B	A(0), G(2), B(3), F-7, C-10	***A1***
4	A, G, B, F	A(0), G(2), B(3), F(7), C-9, E-12	
5	A, G, B, F, C	A(0), G(2), B(3), F(7), C(9), E-10, D-15	***A1***
6	A, G, B, F, C, E	A(0), G(2), B(3), F(7), C(9), E(10), D-14	
7	A, G, B, F, C, E, D	A(0), G(2), B(3), F(7), C(9), E(10), D(14)	***A1***

Note: In both (i) and (ii) accept the tabular method including back tracking or labels by the vertices on a graph.

Note: Award ***M1A1A1A1A0A0*** if final labels are correct but intermediate ones are not shown.

(ii) minimum weight path is ABFCED ***A1***
minimum weight is 14 ***A1***

Note: Award the final two ***A1*** marks whether or not Dijkstra's Algorithm is used.

[8 marks]

Total [12 marks]

3. (a) the equation can be written as

$(3n+3)^2 = n^3 + 3n^2 + 3n + 1$ ***M1A1***

any valid method of solution giving $n = 8$ ***(M1)A1***

Note: Attempt to change at least one side into an equation in n gains the ***M1***.

[4 marks]

(b) **METHOD 1**

as decimal numbers,

$(33)_8 = 27$, $(1331)_8 = 729$ ***A1A1***

converting to base 7 numbers,

$27 = (36)_7$ ***A1***

7)729 ***M1***
7)104(1
7) 14(6
7) 2(0
7) 0(2

therefore $729 = (2061)_7$ ***A1***

the required equation is

$36^2 = 2061$ ***A1***

METHOD 2

as a decimal number, $(33)_8 = 27$ ***A1***

converting to base 7,

$27 = (36)_7$ ***A1***

multiplying base 7 numbers

$$\begin{array}{r} 36 \\ \times\, 36 \\ \hline 1440 \\ 321 \\ \hline 2061 \end{array}$$

1440 ***M1***
321 ***A1***
2061 ***A1***

the required equation is

$36^2 = 2061$ ***A1***

Note: Allow ***M1*** for showing the method of converting a number to base 7 regardless of what number they convert.

[6 marks]

Total [10 marks]

4. (a) evaluating the adjacency matrix to the fifth power ***(M1)***
number of walks $=14$ ***A2***

[3 marks]

(b) number of edges in $G=5$ ***A1***

number of edges in $G'=\begin{pmatrix}5\\2\end{pmatrix}-5$ ***(M1)***

$=5$ ***A1***

Note: Allow listing of edges in G' or drawing graphs.

[3 marks]

(c) (i) the adjacency matrix of G' is

	B	D	A	C	E
B	0	1	0	1	1
D	1	0	0	0	0
A	0	0	0	1	0
C	1	0	1	0	1
E	1	0	0	1	0

A4

Note: Award ***A3*** for one error, ***A2*** for two errors, ***A1*** for three errors and ***A0*** for more than three errors.

(ii) it follows that G and G' are isomorphic because the adjacency matrices of G and G' are identical ***R1***

Note: Or equivalent comprehensive explanation.

[5 marks]

(d) let H have e edges ***M1***

number of edges in $H'=\begin{pmatrix}6\\2\end{pmatrix}-e=15-e$ ***A1***

for an isomorphism to exist, these must be equal: ***M1***
$e=15-e\Rightarrow e=7.5$ ***A1***
which is impossible so no isomorphism ***AG***

[4 marks]

Total [15 marks]

5. (a) using Fermat's little theorem,

$k^p \equiv k \pmod p$ ***(M1)***

therefore,

$\sum_{k=1}^{p} k^p \equiv \sum_{k=1}^{p} k \pmod p$ ***M1***

$\equiv \frac{p(p+1)}{2} \pmod p$ ***A1***

$\equiv 0 \pmod p$ ***AG***

since $\frac{(p+1)}{2}$ is an integer (so that the right-hand side is a multiple of p) ***R1***

[4 marks]

(b) using the alternative form of Fermat's little theorem,

$k^{p-1} \equiv 1 \pmod p$, $1 \le k \le p-1$ ***A1***

$k^{p-1} \equiv 0 \pmod p$, $k = p$ ***A1***

therefore,

$\sum_{k=1}^{p} k^{p-1} \equiv \sum_{k=1}^{p-1} 1 \ (+0) \pmod p$ ***M1***

$\equiv p-1 \pmod p$ ***A1***

(so $n = p-1$)

Note: Allow first ***A1*** even if qualification on k is not given.

[4 marks]

Total [8 marks]